# Quantum Fair Machine Learning

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## QFML Results

Presentation of inaugural results in quantum fair machine learning (**QFML**):

- presents fair machine learning in quantum context
- defines *quantum statistical parity* and *quantum Lipschitz fairness*
- demonstrates use of Grover's amplitude amplification algorithm to achieve statistical parity for quantum algorithms Hilbert spaces $\mathcal{H} = \oplus \mathcal{H}_i$

## Motivation

Quantum computing is a major emerging branch of computing offering potential exponential increases in computational power. Classically infeasible (due to resource constraints) computations may become feasible on a quantum computer & certain intractable problems may be able to be solved:

- as such, the use of quantum algorithms will have ethical consequences. Fair machine learning, which considers how to ethically constrain computation is undeveloped for quantum computing. QFML is therefore well-motivated.
- differences between quantum and quantum computation mean different techniques must be used to impose ethical constraints on how computations occur and outcomes.

## Quantum Information Processing

The key differences between quantum and classical computation of relevance to fair machine learning are [1]:

- *State space.* Quantum systems are described by normalized vectors in Hilbert spaces $|\psi\rangle \in \mathcal{H}$. Vectors are usually two-level qubit systems $|\psi\rangle = a|0\rangle + b|1\rangle$. We also represent states as density operators $\rho = \langle\psi|\,|\psi\rangle$.
- *Superpositions.* Unlike classical bits, the system can be in a superposition of basis states $|0\rangle, |1\rangle$ with probabilities $|a|^2, |b|^2$ respectively. $a, b \in \mathbb{C}$ are called amplitudes. Information is encoded either in amplitudes or basis vectors.
- *Evolution.* Quantum systems and vectors evolve unitarily according to *Schrodinger's equation*:

$$i\hbar \frac{d\,|\psi(t)\rangle}{dt} = H(t)\,|\psi(t)\rangle$$

- *Measurement.* Probability distributions for $|\psi\rangle$ are obtained via measurement operators $M_i$ which yield measurements $m_i$ with probability $\langle\psi|M_i^\dagger M_i|\psi\rangle$. After measurement, the quantum system collapses into post-measurement state, destroying superpositions. Measurement can partition the Hilbert space into disjoint subspaces $\mathcal{H} = \oplus \mathcal{H}_i$, so can act as a classifier.

## Quantum Fairness

*Classification.* Meeting a classification criteria is equivalent to $|\psi\rangle$ residing in the subspace $\mathcal{H}_i$ corresponding to the measurement operator $M_i$ that yields measurement (classification) $m_i$. Different fairness criteria are then represented by different measurement operators which partition the Hilbert space $\mathcal{H}$.

### Definition: Quantum Fairness

*Definition: Quantum Fairness.* Given a suitable POVM $\{E_m\} = \{M_m^\dagger M_m\}$, vectors in $\mathcal{H}$ and quantum state $|\psi\rangle \in \mathcal{H}$ (i.e. $\rho$), the POVM partitions $\mathcal{H}$ (and states) into (possibly disjoint) subspaces $\mathcal{H}_m \in \mathcal{H}_m$. A state $|\psi\rangle$ satisfies quantum fairness with respect to operators $E_m$ that partition the Hilbert space if $|\psi\rangle$ is equally likely to reside in each subspace $\mathcal{H}_m$, that is:

$$\langle\psi|M_m^\dagger M_m|\psi\rangle = \langle\psi|M_n^\dagger M_n|\psi\rangle \quad m \neq n$$
$$\mathrm{tr}(\rho M_m) = \mathrm{tr}(\rho M_n)$$

This general definition of fairness stipulates that $|\psi\rangle$ has an equal probability of residing within each subspace $\mathcal{H}_m$. Meeting a classification $m$ is equivalent to $|\psi\rangle$ residing in the partition $\mathcal{H}_m$ associated with the measurement operator $E_m$ that yields measurement (classification) $m$.

### Definition: Quantum Lipschitz Fairness

*Definition: Quantum Lipschitz Fairness.* We are given a set of input states $\rho_i = \rho_i(t=0)$ and unitary quantum algorithm $\mathcal{A}(t)$ evolving the state to output state after time $t$ given by $\rho_i' = \mathcal{A}(t)^\dagger \rho_i \mathcal{A}(t)$. The quantum equivalent of input metrics $d_X$ and output metrics $d_Y$ are quantum metrics $D_X, D_Y$ such as trace distance $D(\rho_i, \rho_j) = \frac{1}{2}\mathrm{tr}|\rho_i - \rho_j|$, with Lipschitz constraint:

$$D_Y(\rho_i', \rho_j') \leq K(D_X(\rho_i, \rho_j)) \qquad (1)$$

where $D$ is a quantum metric described above, such as trace distance. Other metrics, such as quantum relative entropy or comparison of expectation values are also possible (see paper).

## Example: Amplitude amplification for statistical parity

We basis encode three features of individuals $(x_1, x_2, x_3)$ into three qubits (see Table 1). Our initial quantum state is an equal superposition:

$$|\psi\rangle = \frac{1}{2^n}\sum_{i=1}^{2^n} |x_i^n\rangle$$

We apply a quantum algorithm $\mathcal{A}$ that optimises for some objective. The state evolves to :

$$|\psi\rangle = \sum_{i=1}^{2^n} c_i\,|x_i^n\rangle \qquad c_i \in \mathbb{C}$$

| $i$ (index) | $x_1$ (protected) | $x_2$ | $x_3$ | $\lvert x_i^n\rangle$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | $\lvert 111\rangle$ |
| 2 | 1 | 1 | 0 | $\lvert 110\rangle$ |
| 3 | 1 | 0 | 1 | $\lvert 101\rangle$ |
| 4 | 1 | 0 | 0 | $\lvert 100\rangle$ |
| 5 | 0 | 1 | 1 | $\lvert 011\rangle$ |
| 6 | 0 | 1 | 0 | $\lvert 010\rangle$ |
| 7 | 0 | 0 | 1 | $\lvert 001\rangle$ |
| 8 | 0 | 0 | 0 | $\lvert 000\rangle$ |

Table 1:Basis encoding for three qubits

After application of quantum optimisation algorithm $\mathcal{A}$, $c_i$ are non-uniform & statistical parity is not met. Statistical parity based on the first feature requires an equiprobability of outcome (0 or 1) from measuring the first qubit i.e. we want $Pr(|\psi\rangle\,|x_1 = 0) \approx Pr(|\psi\rangle\,|x_1 = 1)$.

We apply amplitude amplification methods [2, 3] which amplify amplitudes $c_i$ of quantum states until statistical parity criteria are met. To do so we require the existence of an oracle (unitary operator) that partitions $\mathcal{H} = \oplus_m \mathcal{H}_m$ according to measurement outcomes (classifications) $m$ (in our case, binary on $x_1$).
We then define normalised states:

$$|\psi_1\rangle = \frac{1}{\sqrt{M}}\sum_{x_1=1} |x^m\rangle$$
$$|\psi_0\rangle = \frac{1}{\sqrt{N-M}}\sum_{x_1=0} |x^m\rangle$$

Our state $|\psi\rangle$ may be expressed using these two states as a basis such that:

$$|\psi\rangle = \sqrt{\frac{M}{N}}\,|\psi_1\rangle + \sqrt{\frac{N-M}{N}}\,|\psi_0\rangle$$

To achieve the amplification sought via a rotation of $\theta$, we define a unitary operator (so as to preserve quantum coherences and probability measure) $Q(\psi, P) = S_\psi S_\chi$ using the operators:

$$S_\psi = 2\,|\psi\rangle\langle\psi| - \mathbb{I}$$
$$S_\chi = 2P - \mathbb{I}$$

he product of these two reflections is a rotation through Hilbert space:

$$|\psi\rangle = \sin(\theta/2)\,|\psi_1\rangle + \cos(\theta/2)\,|\psi_0\rangle \qquad (2)$$

We select our oracle as the projector:

$$P = |1\rangle\langle 1| \otimes \mathbb{I} \otimes \mathbb{I} \qquad (3)$$

Applying $Q$ operator $k$ timesthen results in:

$$Q^k\,|\psi\rangle = \sin\left(\frac{2k+1}{2}\theta\right)|\psi_1\rangle + \cos\left(\frac{2k+1}{2}\theta\right)|\psi_0\rangle \qquad (4)$$

This has the effect of rotating $|\psi\rangle$ by $\theta$ (geometrically, counter-clockwise) so as to increase the amplitude of $|\psi_1\rangle$. The probability of measuring a state in $\mathcal{H}_1$ (i.e. the probability that $|\psi\rangle \equiv |\psi_1\rangle$) is then:

$$Pr(|\,|\psi_1\rangle\,|) = \sin^2([(2k+1)/2]\theta) \qquad (5)$$

By applying $Q$ a sufficient number of times, the probabilities of measuring a state in $\mathcal{H}_0$ and $\mathcal{H}_1$ can be approximately equalised.

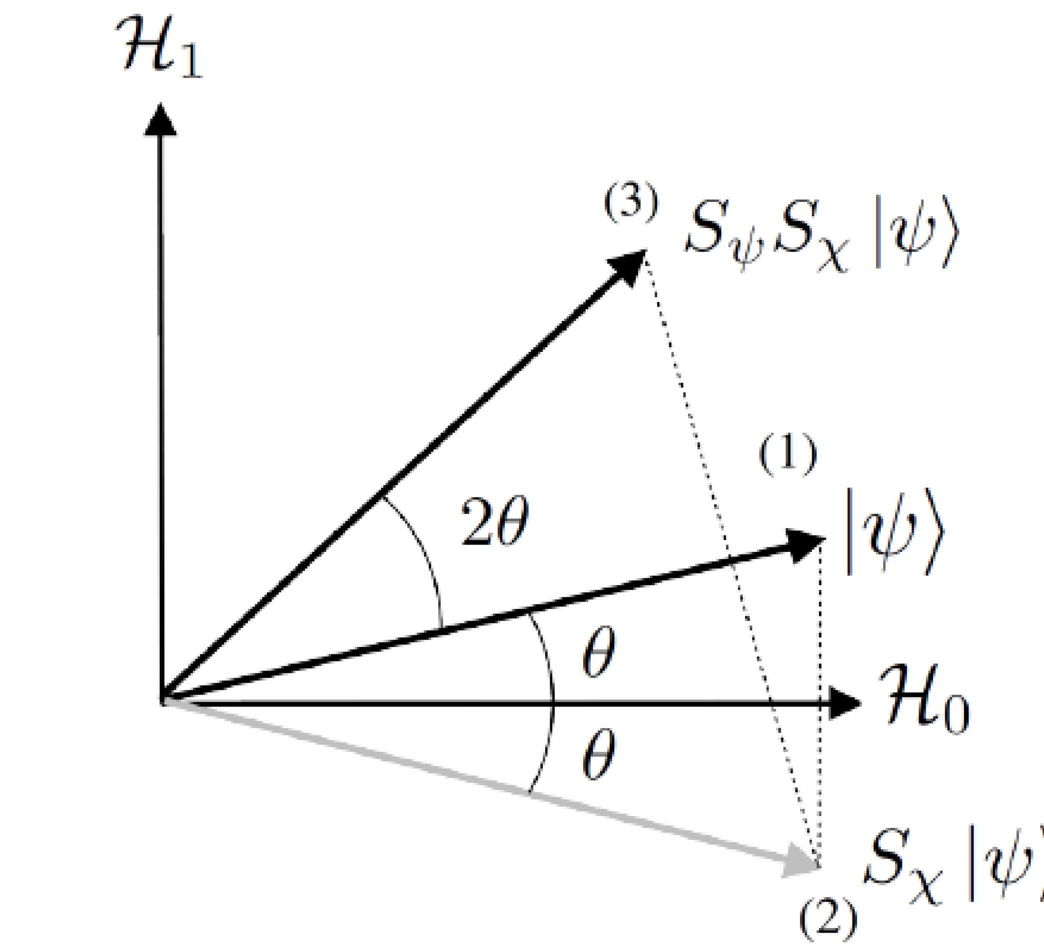


Figure 1:$|\psi\rangle$ is rotated by amplitude amplification to reside equiprobably within $\mathcal{H}_1, \mathcal{H}_2$

We use the above procedure to achieve quantum *$\epsilon$-statistical parity* (namely equal probability within $\epsilon$) such that:

$$|\,\langle\psi_1|\psi|\psi_1\rangle - \langle\psi_0|\psi|\psi_0\rangle\,| = \epsilon \qquad (6)$$

which can be expressed as:

$$|0.5 - \epsilon| = \sin^2((2m+1)\theta) \qquad (7)$$

Such statistical parity can be approximated by applying the amplitude amplification operator $Q$ iterative $m$ times where $m$ is given by:

$$m = \left|\frac{\arcsin\sqrt{(|0.5-\epsilon|)}}{2\theta} - \theta\right| \qquad (8)$$

Future research: (a) formalising quantum analogues of existing techniques in FML, (b) exploring QFML in noisy contexts, especially in dissipative open quantum systems, (c) examining how fairness outcomes and computation differs as a result of using quantum-specific resources, such as entanglement and (d) the role of cryptographic and quantum analogues of differential privacy for satisfying fairness criteria for quantum computations.

## References


[1] M.A. Nielsen and I.L. Chuang.
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